DOCUMENT RESUME

ED 038 305 24 SE 008 252

AUTHOR	Shepler, Jack L.
TITLE	A Study of Parts of the Development of a Unit in Probability and Statistics for the Elementary School, (Part 4) and Appendices B, C, and D.
INSTITUTION	Wisconsin Univ., Madison. Research and Development Center for Cognitive Learning.
SPONS AGENCY	Office of Education (DHEW), Washington, D.C. Bureau of Research.
REPORT NO	TR-105
BUREAU NO	BR-5-0216
PUB DATE	Nov 69
CONTRACT	OEC-5-10-154
NOTE	60p.
EDRS PRICE	EDRS Price MF-$0.50 HC-$3.10
DESCRIPTORS	*Curriculum Development, Doctoral Theses, *Elementary School Mathematics, Grade 6, *Instruction, Mathematics Education, *Research, Statistics

ABSTRACT

Reported is research conducted as a part of the Project on Analysis of Mathematics Instruction. The study had two main purposes: to test the feasibility of teaching topics in probability and statistics to a class of sixth grade students; and to construct a set of instructional materials and procedures in probability and statistics for sixth graders. A unit of instruction was prepared and the order in which behavioral objectives were to be taught was determined from a content outline and a task analysis. The results of the study support the feasibility of teaching most of the topics covered in the unit to average and above average sixth graders. The study also lends support to the use of the systems developmental model employed for developing curriculum materials. Part IV contains Appendices B, C, and D which include the testing instrument used for pre- and post-testing, tables presenting data from the formative evaluation, and background data of the students. (FL)

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ort No. 105 (Part IV) Appendices B, C, and D

A STUDY OF PARTS OF THE DEVELOPMENT OF A UNIT IN PROBABILITY AND STATISTICS FOR THE ELEMENTARY SCHOOL

Report from the Project on
Analysis of Mathematics Instruction

SE 008 252

U.S. Office of Education Center No. C-03 Contract OE 5-10-154

ED038305

MAR 30 1970

Technical Report No. 105 (Part IV) Appendices B, C, and D

A STUDY OF PARTS OF THE DEVELOPMENT OF A UNIT IN PROBABILITY AND STATISTICS FOR THE ELEMENTARY SCHOOL

Report from the Project on
Analysis of Mathematics Instruction

By Jack L. Shepler

Thomas A. Romberg, Assistant Professor of Curriculum & Instruction
Chairman of the Examining Committee

Thomas A. Romberg, Principal Investigator

Wisconsin Research and Development
Center for Cognitive Learning
The University of Wisconsin
Madison, Wisconsin
November 1969

This Technical Report is a doctoral dissertation reporting research supported by the Wisconsin Research and Development Center for Cognitive Learning. Since it has been approved by a University Examining Committee, it has not been reviewed by the Center. It is published by the Center as a record of some of the Center's activities and as a service to the student. The bound original is in The University of Wisconsin Memorial Library.

Published by the Wisconsin Research and Development Center for Cognitive Learning, supported in part as a research and development center by funds from the United States Office of Education, Department of Health, Education, and Welfare. The opinions expressed herein do not necessarily reflect the position or policy of the Office of Education and no official endorsement by the Office of Education should be inferred.

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STATEMENT OF FOCUS

The Wisconsin Research and Development Center for Cognitive Learning focuses on contributing to a better understanding of cognitive learning by children and youth and to the improvement of related educational practices. The strategy for research and development is comprehensive. It includes basic research to learning and about the processes of instruction, and the subsequent development of research-based instructional materials, many of which are designed for use by teachers and others for use by students. These materials are tested and refined in school settings. Throughout these operations behavioral scientists, curriculum experts, academic scholars, and school people interact, insuring that the results of Center activities are based soundly on knowledge of subject matter and cognitive learning and that they are applied to the improvement of educational practice.

This Technical Report is from Phase 2 of the Project on Prototypic Instructional Systems in Elementary Mathmatics in Program 2. General objectives of the Program are to establish rationale and strategy for developing instructional systems, to identify sequences of concepts and cognitive skills, to develop assessment procedures for those concepts and skills, to identify or develop instructional materials associated with the concepts and cognitive skills, and to generate new knowledge about instructional procedures. Contributing to the Program objectives, the Mathematics Project, Phase 1, is developing and testing a televised course in arithmetic for Grades 1-6 which provides not only a complete program of instruction for the pupils but also inservice training for teachers. Phase 2 has a long-term goal of providing an individually guided instructional program in elementary mathematics. Preliminary activities include identifying instructional objectives, student activities, teacher activities materials, and assessment procedures for integration into a total mathematics curriculum. The third phase focuses on the development of a computer system for managing individually guided instruction in mathematics and on a later extension of the system's applicability.

ACKNOWLEDGEMENTS

Many people have contributed to the completion of this study. However, I would like to thank in particular Dr. Thomas A. Romberg for the guidance and knowledge he imparted to me during my three years of graduate study. Appreciation is also extended to the other members of the reading committee, Drs. John G. Harvey and Robert E. Davidson for their help and comments concerning the study.

The author is deeply grateful to the staff of the Research and Development Center for their support in providing a teacher, technical advice, and clerical help. The author is particularly grateful to Mrs. Carolyn Gornowicz who taught the unit.

My deepest appreciation is expressed to my wife, Gloria, for her typing of rough drafts, proofreading, etc; but more importantly for her understanding and encouragement.

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ABSTRACT

From a content outline and a task analysis the behavioral objectives for a unit of instruction in probability and statistics for sixth-grade students and the order in which objectives would be taught were determined. An instructional analysis of the unit was undertaken to select or develop materials and procedures for teaching the unit.

Data from a pilot study conducted in the fall of 1969 were used to identify a set of nine lessons that could be formatively evaluated to test the feasibility of the instructional analysis. The lessons were used to teach a class of 25 sixth-grade students of average to above average ability. The topics developed through experiments, games and exercises were subjective probability notions, empirical probability, counting techniques, a priori probability including simple and compound events, and comparison of two events using probability.

On the basis of the overall pretest and posttest the instructional treatment was generally successful. The pretest percentage was 37.9% and the posttest percentage was 92.8% with all 72 items successful for 11 of the 14 measured objectives. Instruction was unsuccessful in getting students to specify the estimated probability; number the outcomes of an event; and estimate the probability successful for these three objectives because of a lack of stress and practice. Two learning hierarchies were also tested. One hierarchy was validated and the other was not. The results of the study support the feasibility of teaching most of the included topics in probability and statistics to average and above average sixth-grade students given high quality of teaching. The study lends support to the use of the systems developmental model employed in this study for developing curriculum materials for the schools, especially when used in conjuction with Bloom's "Mastery Learning" techniques.

APPENDIX B

TESTING INSTRUMENT USED FOR PRETESTING AND POSTTESTING

Name _____

Instructions

Place your name in the blank at the left.
(Do not turn this page until you are told)

The questions you are about to answer were written to find out how much you know about a certain branch of mathematics that deals with "luck" or "probability". Read each question very carefully and write your answers in the answer blanks provided. If the question is a multiple choice question place the letter you think is correct in the blank provided.

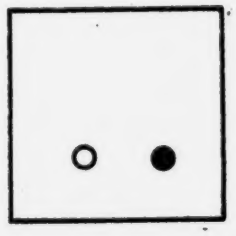
There are different kinds of questions. Some questions ask, "what is the probability of ...". Others ask you to tell "how many" or "which game gives you the best chance of winning". Be sure you answer each question in the proper way. Read carefully.

Since you have not studied this topic before, there will be questions for which you do not know the answers. Leave such questions blank. Do not guess.

It is assumed that each game or activity is to be played fairly.

You may take as much time as you need to do the test. Work carefully. You may use the blank space by a question to do your scratch work.

For problems (1 - 8), a marble is picked from the box below one or more times. After a marble is picked the color is recorded and the marble is returned to the box. Classify the statements about chance as being certain to happen, uncertain as to whether it could happen, or impossible to happen. Place the letter that indicates your choice in the blank.



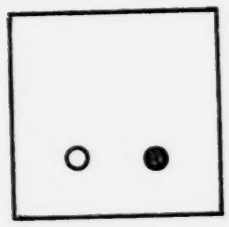
_____ 1. The chances of getting a black or white marble in picking from the box are (a) certain (b) uncertain (c) impossible.

_____ 2. The chances of getting a striped marble in picking from the box are (a) certain (b) uncertain (c) impossible.

_____ 3. The chances of getting 50 black marbles in picking from a box 50 times are (a) certain (b) uncertain (c) impossible.

_____ 4. The number of black in picking 100 times from the box may be 0 or 100 or anything in between is (a) certain (b) uncertain (c) impossible.

The following statements are true or false. If you think a statement is true, put a T in the blank after the statement. If you think the statement is <u>not</u> <u>true</u>, put an F in the blank. The problems pertain to the picture at the right.



_____ 5. It is possible to pick 200 times without getting a single black marble.

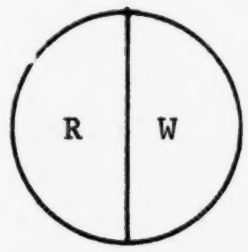
_____ 6. If we pick a marble 1000 times it is likely that we will get between 400 and 600 blacks.

_____ 7. You have picked from the box 5 times and have gotten 5 black marbles. It is more likely that the next pick will be white than black.

_____ 8. You have picked from the box 30 times and have gotten 30 white marbles in a row. You are almost certain to get a black marble in the next pick from the box.

Multiple Choice

_____ 1. If you spin the fair spinner at the right 10 times and get 10 red, the chances of getting white on the 11th spin are



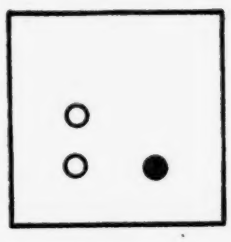
(a) almost certain

(b) 1/2

(c) certain

(d) none of the above

_____ 2. You draw a marble without looking, from the box below 20 times and get 20 black marbles. Each time you pick a marble you put it back. The chances of getting a black marble on the 21st draw are



(a) impossible

(b) almost impossible

(c) 1/3

(d) none of the above

Identify whether the following fractions are equal or if one is larger than the other.

_____ 1. 2/6, 1/3

(a) 2/6 = 1/3 (b) 2/6 is greater than 1/3

(c) 1/3 is greater than 2/6

_____ 2. 1/2, 1/3

(a) 1/2 = 1/3 (b) 1/2 is greater than 1/3

(c) 1/3 is greater than 1/2

_____ 3. 2/8, 3/9

(a) 2/8 = 3/9 (b) 2/8 is greater than 3/9

(c) 3/9 is greater than 2/8

_____ 4. 1/2, 3/6

(a) 1/2 = 3/6 (b) 1/2 is greater than 3/6

(c) 3/6 is greater than 1/2

_____ 5. 1/5, 2/10

(a) 1/5 = 2/10 (b) 1/5 is greater than 2/10

(c) 2/10 is greater than 1/5

_____ 6. 2/10, 1/8

(a) 2/10 = 1/8 (b) 2/10 is greater than 1/8

(c) 1/8 is greater than 2/10

_____ 7. 2/3, 2/4

(a) 2/3 = 2/4 (b) 2/3 is greater than 2/4

(c) 2/4 is greater than 2/3

_____ 8. 1/4, 3/12

(a) 1/4 = 3/12 (b) 1/4 is greater than 3/12

(c) 3/12 is greater than 1/4

_____ 9. 3/8, 2/5

(a) 3/8 = 2/5 (b) 3/8 is greater than 2/5

(c) 2/5 is greater than 3/8

_____10. 1/4, 7/16

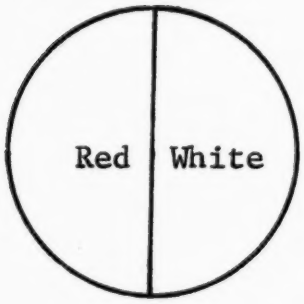
(a) 1/4 = 7/16 (b) 1/4 is greater than 7/16

(c) 7/16 is greater than 1/4.

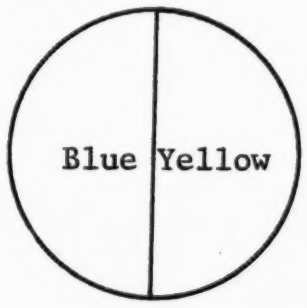
The following set of problems ask you to choose the game which gives you the best chance of winning. If the chances are the same you are to identify that it doesn't make any difference. Place an X in the blank to the left of the choice which you think is correct. Give a short explanation as to why you gave the answer you did.

1. Which game gives you the best chance of winning or doesn't it make any difference?

First Spinner



Second Spinner



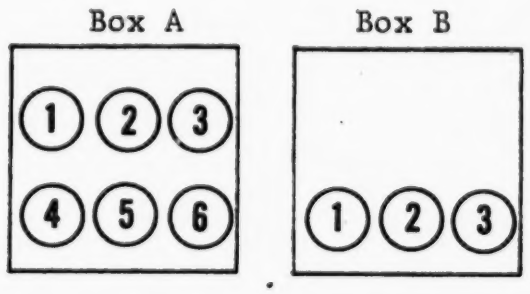
_____ Game 1 You spin the first spinner once and you win if you get red.

_____ Game 2 You spin both spinners once and you win if you get white and yellow, or white and blue.

_____ It doesn't make any difference.

Why?

2. Which game gives you the best chance of winning or doesn't it make any difference?



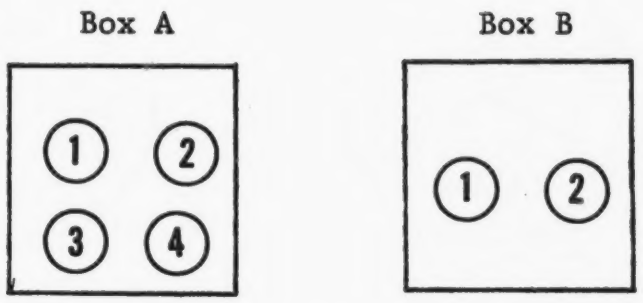
_____ Game 1 You pick a chip from Box A and you win if you pick the chip with "2" on it.

_____ Game 2 You pick twice from Box B (without putting the first chip back). You win if you get a "1" on the first pick and a "2" on the second pick.

_____ It doesn't make any difference.

Why?

3. Which game gives you the best chance of winning or doesn't it make any difference?



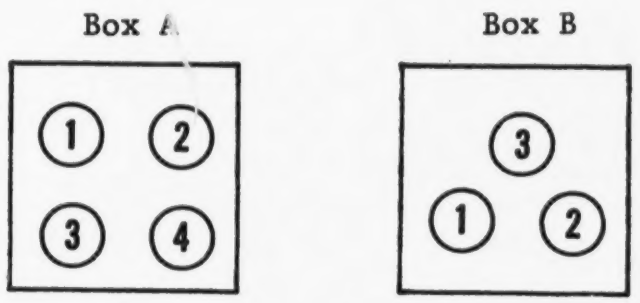
_____ Game 1 You pick a chip from Box A and you win if you pick the chip with "3" on it.

_____ Game 2 You pick one chip from Box A and one chip from Box B. You win if you get a "3" from Box A and a "2" from Box B.

_____ It doesn't make any difference.

Why?

4. Which game gives you the best chance of winning or doesn't it make any difference?



_____ Game 1 You pick a chip from Box A and you win if you pick the chip with "1" on it.

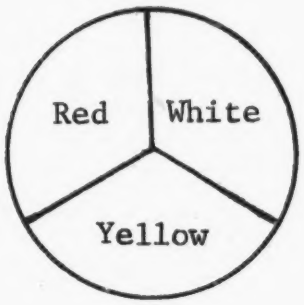
_____ Game 2 You pick one chip from Box A and one chip from Box B and find the sum of the numbers on the chips. You win if the sum is 4 .

_____ It doesn't make any difference.

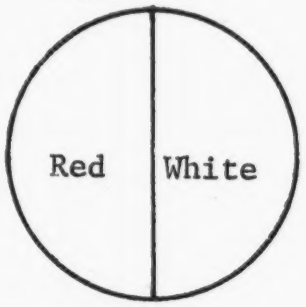
Why?

5. Which game gives you the best chance of winning or doesn't it make any difference?

Spinner A



Spinner B



_____ Game 1 You spin Spinner A once and you win if you get white.

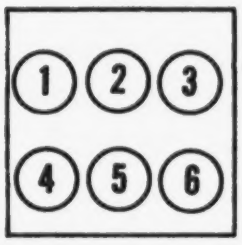
_____ Game 2 You spin Spinner B twice. You win if you get Red on the first spin and White on the second spin.

_____ It doesn't make any difference.

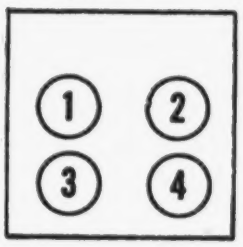
Why?

6. Which game gives you the best chance of winning or doesn't it make any difference?

Box A



Box B



_____ Game 1 You pick a chip from Box A and you win if you pick the chip with "2" on it.

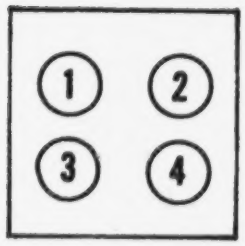
_____ Game 2 You pick twice from Box B (without putting the first chip you pick back). You win if you pick a "2" first and a "4" on the second pick.

_____ It doesn't make any difference.

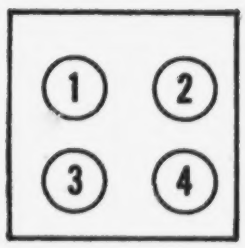
Why?

7. Which game gives you the best chance of winning or doesn't it make any difference?

Box A



Box B



_____ Game 1 You pick one chip from Box A and you win if you get a chip with a "4" on it.

_____ Game 2 You pick one chip from Box A and one from Box B and find the sum of the numbers on the chips. You win if you get a sum of 4 or 5 .

_____ It doesn't make any difference.

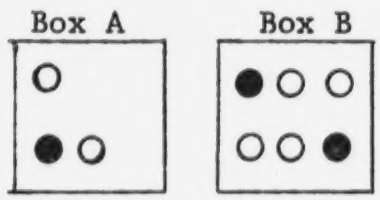
Why?

The following boxes contain black and white marbles. To play this game you pick a marble from one of the two boxes. You win if you choose a black marble.

If you can play this game only once, do you have a better chance of winning if you pick from Box A or Box B, or doesn't it make any difference?

For each question, place an "X" in the blank at the right that shows your choice.

1.

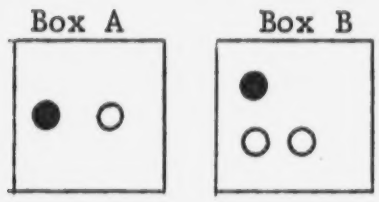


_____ Box A

_____ Box B

_____ It doesn't make any difference

2.

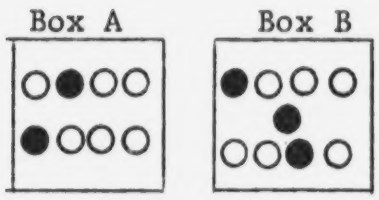


_____ Box A

_____ Box B

_____ It doesn't make any difference

3.

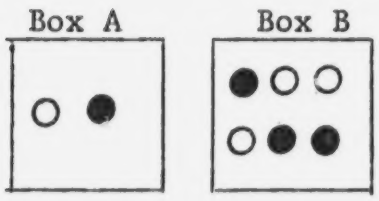


_____ Box A

_____ Box B

_____ It doesn't make any difference

4.

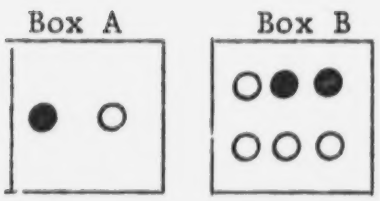


_____ Box A

_____ Box B

_____ It doesn't make any difference

5.

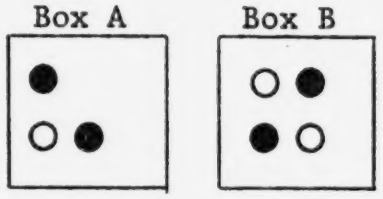


_____ Box A

_____ Box B

_____ It doesn't make any difference

6.

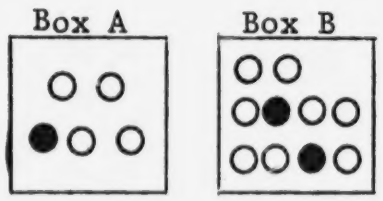


_____ Box A

_____ Box B

_____ It doesn't make any difference

7.

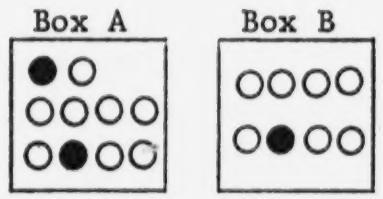


_____ Box A

_____ Box B

_____ It doesn't make any difference

8.

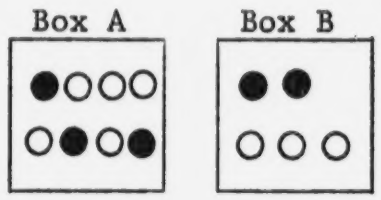


_____ Box A

_____ Box B

_____ It doesn't make any difference

9.

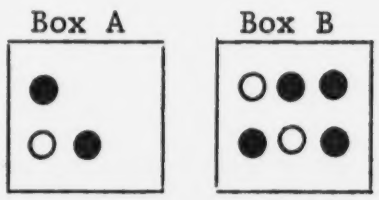


_____ Box A

_____ Box B

_____ It doesn't make any difference

10.

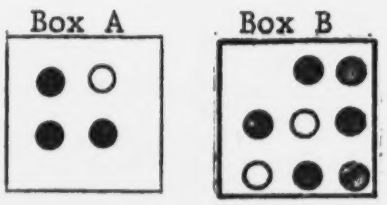


_____ Box A

_____ Box B

_____ It doesn't make any difference

11.

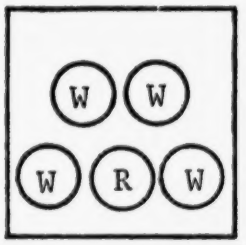


_____ Box A

_____ Box B

_____ It doesn't make any difference

For the following problems a marble is drawn without looking from a bag containing 4 white marbles and 1 red marble.



_____ 1. How many possible outcomes are there?

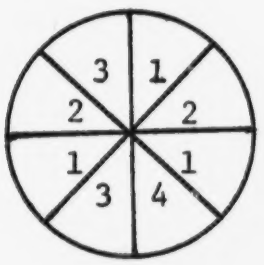
_____ 2. What is the probability of getting a red marble?

_____ 3. What is the probability of getting a white marble?

_____ 4. What is the probability of getting a red or white marble?

_____ 5. What is the probability of getting a green marble?

For the following problems you spin the spinner below one time.



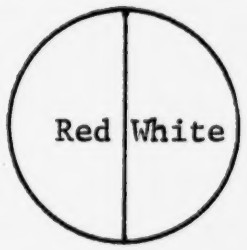
_____ 1. How many possible outcomes are there?

_____ 2. How many ways can you get a "1"?

_____ 3. What is the probability of getting a "1"?

_____ 4. What is the probability of getting a "4"?

For the following problem you spin the spinner at the right <u>two times</u>.

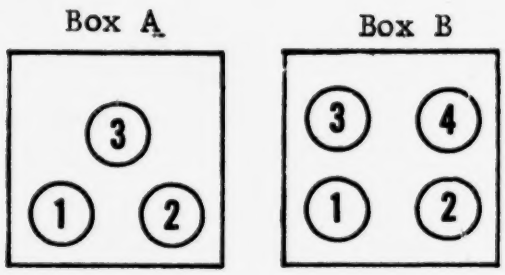


_____ 1. How many possible outcomes are there?

_____ 2. How many ways can one get white on the first spin and white on the second spin?

_____ 3. What is the probability of getting red on the first spin and red on the second spin?

_____ 4. What is the probability of getting red on the first spin and either color on the second spin?



For the following problems you pick a numbered chip without looking from Box A and one from Box B.

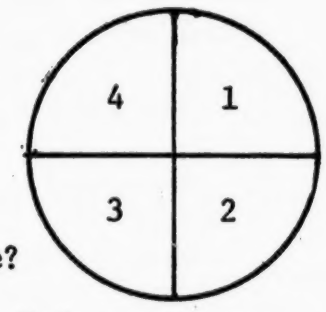
_____ 1. How many possible outcomes are there?

_____ 2. How many ways can one get a sum of "4" when the two numbers on the chips are added together?

_____ 3. What is the probability of getting "1" from Box A and "1" from Box B?

_____ 4. What is the probability of getting an outcome whose sum is equal to 5?

_____ 5. What is the probability of getting an outcome whose sum is equal to 8?



For the following problems you spin the spinner at the right <u>two</u> times.

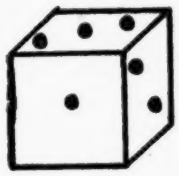
_____ 1. How many possible outcomes are there?

_____ 2. What is the probability of getting a "2" on the first spin and a "4" on the second spin?

_____ 3. What is the probability of getting an outcome whose <u>sum</u> is equal to <u>2</u>?

_____ 4. What is the probability of getting an outcome whose <u>sum</u> is equal to <u>6</u>?

_____ 5. What is the probability of getting an outcome whose <u>sum</u> is less than <u>10</u>?

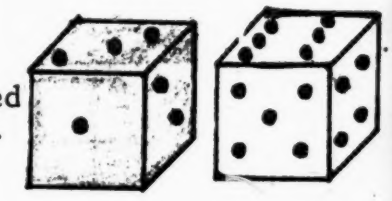
For the following problem a die is thrown  and the face which turns up is recorded.

_____ 1. How many possible outcomes are there?

_____ 2. How many ways can one get a "3" or a "5"?

_____ 3. What is the probability of getting a "2"?

_____ 4. What is the probability of getting a "3" or a "5"?



For the following problem, two dice are thrown, one red and one white. The sum of the faces turning up is recorded.

_____ 1. How many ways can all the sums be gotten?

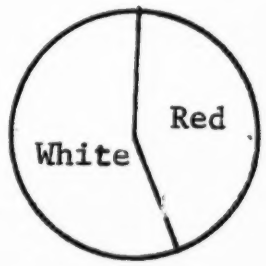
_____ 2. How many ways can one get the sum 2 or 3 ?

_____ 3. What is the probability of getting the sum 12 ?

_____ 4. What is the probability of getting the sum 11 ?

_____ 5. What is the probability of getting a sum of 2 or 3 ?

_____ 1. In 6000 spins of the spinner at the right Bob gets 2653 reds. What is the estimated probability of getting a red on the next spin?



_____ 1. Bob tossed a thumbtack 9,000 times with the following results:

6003 - point up
2997 - point down

Which of the following statements could Bob make?

a. The chances of the thumbtack pointing down is about 1/2.
b. The chances of the thumbtack pointing down is about 1/3.
c. The chances of the thumbtack pointing down is exactly 2997/6003.
d. The chances of the thumbtack pointing down is exactly 2997/9000.
e. Bob can make no statement at all about the chances of the thumbtack pointing down.

APPENDIX C

TABLES PRESENTING DATA FROM THE FORMATIVE EVALUATION

Tables 16 through 29 present the pretest and posttest data for each of the measured behavioral objectives. The first column of each table lists as an ordered pair the part and the item number for the test (see). For example (1,5) means that the item is Item 5 in Part I of the test in Appendix B. (E.g. (1, 5) is

> "5. It is possible to pick 200 times without getting a single black marble."

The number of correct responses on item (1, 5) for the pretest was 4 (out of 25) and for the posttest was 22 (out of 25).

The total number of correct responses for the items is listed under "Total." With regard to items measuring a behavioral objective in a one dimensional (1D) and a two dimensional sample space (2D), the data is presented separately under the headings "1D" and "2D." The "Total" in this case presents the sum of the correct responses in "1D" and "2D" separately while the "Grand Total" combines the results' of "1D" and "2D" columns together.

One should keep in mind that the total number of possible correct responses for item is 25, the number of children in the study.

TABLE 16

BEHAVIORAL OBJECTIVE 1 -- DISTINGUISH WHETHER AN EVENT IS AN INSTANCE OF CERTAINTY, UNCERTAINTY, OR IMPOSSIBILITY. (5 ITEMS)

Item Number Part	Item	No. of correct responses (Pretest)	No. of Correct responses (Posttest)
(1,	1)	18	25
(1,	2)	23	25
(1,	3)	18	23
(1,	4)	15	24
(1,	5)	14	22
Total		88	119

TABLE 17

BEHAVIORAL OBJECTIVE 2 -- COUNT THE NUMBER OF OUTCOMES OF AN EVENT (5 ITEMS)

Item Number Part	Item	No. of Correct responses (Pretest) 1D	2D	No. of Correct responses (Posttest) 1D	2D
(7,	2)	21		25	
(8,	2)		4		16
(9,	2)		6		22
(11,	2)	7		17	
(12,	2)		1		19
Total		28	11	42	57
Grand Total 1D and 2D		39		99	

TABLE 18

BEHAVIORAL OBJECTIVE 3 -- COUNT THE NUMBER OF POSSIBLE OUTCOMES OF A SAMPLE SPACE (7 ITEMS)

Item Number Part	Item	No. of Correct responses (Pretest) 1D	2D	No. of Correct responses (Posttest) 1D	2D
(6,	1)	11		24	
(8,	1)		1		25
(10,	1)		0		23
(12,	1)		0		22
(7,	1)	12		25	
(9,	1)		0		23
(11,	1)	21		23	
Total		44	1	72	93
Grand Total 1D and 2D		45		165	

TABLE 19

BEHAVIORAL OBJECTIVE 4a -- SPECIFY THE PROBABILITY OF A SIMPLE EVENT
(8 ITEMS)

Item Number Part Item	No. of Correct responses (Pretest) 1D	2D	No. of Correct responses (Posttest) 1D	2D
(6, 2)	7		25	
(8, 3)		0		25
(10, 2)		0		25
(10, 3)		0		25
(12, 3)		0		25
(7, 4)	8		25	
(9, 3)		0		25
(11, 3)	6		24	
Total	21	0	74	125
Grand Total	21		199	

TABLE 20

BEHAVIORAL OBJECTIVE 4b -- SPECIFY THE PROBABILITY OF A COMPOUND EVENT (8 ITEMS)

Item Number Part	Item	No. of Correct responses (Pretest) 1D	2D	No. of Correct responses (Posttest) 1D	2D
(6,	3)	6		25	
(8,	4)		2		23
(10,	4)		1		25
(12,	4)		0		24
(12,	5)		0		25
(7,	3)	8		25	
(9,	4)		0		21
(11,	4)	4		23	
Total		18	3	73	118
Grand Total 1D and 2D		21		191	

TABLE 21

BEHAVIORAL OBJECTIVE 4c -- SPECIFY THE PROBABILITY OF A CERTAIN EVENT (2 ITEMS)

Item Number Part	Item	No. of Correct Response (Pretest 1D	2D	No. of Correct Response (Posttest) 1D	2D
(6,	4)	3		24	
(10,	5)		5		23
Total		3	5	24	23
Grand Total 1D and 2D		8		47	

TABLE 22

BEHAVIORAL OBJECTIVE 4d -- SPECIFY THE PROBABILITY OF THE IMPOSSIBLE EVENT (2 ITEMS)

Item Number Part	Item	No. of Correct responses (Pretest) 1D	2D	No. of Correct responses (Posttest) 1D	2D
(6,	5)	12		25	
(9,	5)		5		24
Total		12	5	25	24
Grand Total (1D and 2D		17		49	

TABLE 23

BEHAVIORAL OBJECTIVE 5 -- SPECIFY THE ORDER OF TWO FRACTIONS BETWEEN 0 AND 1 (10 ITEMS)

Item Number Part	Item	No. of Correct Responses (Pretest)	No. of Correct Responses (Posttest)
(3,	1)	24	25
(3,	2)	22	25
(3,	3)	16	24
(3,	4)	24	25
(3,	5)	24	25
(3,	6)	18	23
(3,	7)	17	24
(3,	8)	22	25
(3,	9)	17	23
(3,	10)	13	21
Total		197	240

TABLE 24

BEHAVIORAL OBJECTIVE 6 -- IDENTIFY THE MOST LIKELY EVENT OF TWO UNEQUALLY LIKELY EVENTS (10 ITEMS)

Item Number Part	Item	No. of correct responses (Pretest) 1D	1D and 2D	No. of correct reponses (Posttest) 1D	1D and 2D
(4,	3)		10		24
(4,	5)		4		23
(4 ,	6)		2		23
(4,	7)		13		20
(5,	2)	15		24	
(5,	3)	18		25	
(5,	5)	17		25	
(5,	6)	15		24	
(5,	8)	13		25	
(5,	9)	13		25	
Total		91	29	148	90
Grand Total (1D plus 1D and 2D)		120		238	

TABLE 25

BEHAVIORAL OBJECTIVE 7 -- IDENTIFY TWO UNEQUALLY LIKELY EVENTS AS BEING EQUALLY LIKELY (8 ITEMS)

Item Number Part	Item	No. of correct responses (Pretest) 1D	1D and 2D	No. of correct responses (Posttest) 1D	1D and 2D
(4,	1)		9		23
(4,	2)		7		22
(4,	4)		4		23
(5,	1)	8		23	
(5,	4)	8		25	
(5,	7)	13		24	
(5,	10)	7		23	
(5,	11)	8		23	
Total Number of Correct Responses		44	20	118	68
Grand Total (1D plus 1D and 2D)		64		186	

TABLE 26

BEHAVIORAL OBJECTIVE 8 -- SPECIFY THE ESTIMATED PROBABILITY OF AN EVENT, GIVEN THE DATA FROM AN EXPERIMENT (1 ITEM)

Item Number Part	Item	No. of correct responses (Pretest)	No. of correct responses (Posttest)
13,	1	2	13

TABLE 27

BEHAVIORAL OBJECTIVE 9 -- IDENTIFY THE LIKELY BOUNDS ON THE FREQUENCY OF AN OUTCOME OF AN EXPERIMENT PERFORMED N TIMES (1 ITEM)

Item Number Part	Item	No. of correct responses (Pretest)	No. of correct responses (Posttest)
(1,	6)	10	25

TABLE 28

BEHAVIORAL OBJECTIVE 10 -- IDENTIFY AN INSTANCE OF THE LAW OF AVERAGES (4 ITEMS)

Item Number Part	Item	No. of correct responses (Pretest)	No. of correct responses (Posttest)
(1,	7)	16	22
(1,	8)	15	22
(2,	1)	10	23
(2,	2)	8	25
Total Number of correct responses		49	92

TABLE 29

BEHAVIORAL OBJECTIVE 11 -- IDENTIFY AN ESTIMATE OF THE PROBABILITY GIVEN A SET OF DATA FROM AN EXPERIMENT (1 ITEM)

Item Number Part	Item	No. of correct responses (Pretest)	No. of correct responses (Posttest)
14,	1	1	7

Tables 30 through 43 present the break-down of the ratio of students achieving a specified test criterion level on the posttest. For example, in Table 30, 24 students out of 25 (96%) scored 80% or better (4/5 or 5/5) on the five items measuring Behavioral Objective 1.

TABLE 30

BEHAVIORAL OBJECTIVE 1 (POSTTEST)

Ratio of Children Reaching Test Criterion	Percentage of Children Reaching Test Criterion	Test Criterion (Ratio Form)	Test Criterion (Percentage Form)
20/25	80	5/5	100
24/25	96	4/5	80
25/25	100	3/5	60

TABLE 31

BEHAVIORAL OBJECTIVE 2 (POSTTEST)

Ratio of Children Reaching Test Criterion	Percentage of Children Reaching Test Criterion	Test Criterion (Ratio Form)	Test Criterion (Percentage Form)
11/25	44	5/5	100
18/25	72	4/5	80
22/25	88	3/5	60
22/25	88	2/5	40
25/25	100	1/5	20

TABLE 32

BEHAVIORAL OBJECTIVE 3 (POSTTEST)

Ratio of Children Reaching Test Criterion	Percentage of Children Reaching Test Criterion	Test Criterion (Ratio Form)	Test Criterion (Percentage Form)
18/25	72	7/7	100
22/25	88	6/7	85.7
25/25	100	5/7	71.4

TABLE 33

BEHAVIORAL OBJECTIVE 4a (POSTTEST)

Ratio of Children Reaching Test Criterion	Percentage of Children Reaching Test Criterion	Test Criterion (Ratio Form)	Test Criterion (Percentage Form)
24/25	96	8/8	100
25/25	100	7/8	87.5

TABLE 34

BEHAVIORAL OBJECTIVE 4b (POSTTEST)

Ratio of Children Reaching Test Criterion	Percentage of Children Reaching Test Criterion	Test Criterion (Ratio Form)	Test Criterion (Percentage Form)
18/25	72	8/8	100
23/25	88	7/8	87.5
25/25	100	6/8	75

TABLE 35

BEHAVIORAL OBJECTIVE 4c (POSTTEST)

Ratio Children Reaching Test Criterion	Percentage of Children Reaching Test Criterion	Test Criterion (Ratio Form)	Test Criterion (Percentage Form)
22/25	88	2/2	100
25/25	100	1/2	50

TABLE 36

BEHAVIORAL OBJECTIVE 4d (POSTTEST)

Ratio Children Reaching Test Criterion	Percentage of Children Reaching Test Criterion	Test Criterion (Ratio Form)	Test Criterion (Percentage Form)
24/25	96	2/2	100
25/25	100	1/2	50

TABLE 3 7

BEHAVIORAL OBJECTIVE 5 (POSTTEST)

Ratio Children Reaching Test Criterion	Percentage of Children Reaching Test Criterion	Test Criterion (Ratio Form)	Test Criterion (Percentage Form)
18/25	72	10/10	100
23/25	92	9/10	90
24/25	96	8/10	80
25/25	100	7/10	70

TABLE 38

BEHAVIORAL OBJECTIVE 6 (POSTTEST)

Ratio Children Reaching Test Criterion	Percentage of Children Reaching Test Criterion	Test Criterion (Ratio Form)	Test Criterion (Percentage Form)
16/25	64	10/10	100
23/25	92	9/10	90
24/25	96	8/10	80
25/25	100	7/10	70

TABLE 39

BEHAVIORAL OBJECTIVE 7 (POSTTEST)

Ratio Children Reaching Test Criterion	Percentage of Children Reaching Test Criterion	Test Criterion (Ratio Form)	Test Criterion (Percentage Form)
15/25	60	8/8	100
23/25	92	7/8	87.5
24/25	96	6/8	75.0
25/25	100	4/8	50.0

TABLE 40

BEHAVIORAL OBJECTIVE 8 (POSTTEST)

Ratio Children Reaching Test Criterion	Percentage of Children Reaching Test Criterion	Test Criterion (Ratio Form)	Test Criterion (Percentage Form)
13/25	52	1/1	100

TABLE 41

BEHAVIORAL OBJECTIVE 9 (POSTTEST)

Ratio Children Reaching Test Criterion	Percentage of Children Reaching Test Criterion	Test Criterion (Ratio Form)	Test Criterion (Percentage Form)
25/25	100	1/1	100

TABLE 42

BEHAVIORAL OBJECTIVE 10 (POSTTEST)

Ratio Children Reaching Test Criterion	Percentage of Children Reaching Test Criterion	Test Criterion (Ratio Form)	Test Criterion (Percentage Form)
21/25	84	4/4	100
22/25	88	3/4	75
24/25	96	2/4	50
25/25	100	1/4	25

TABLE 43

BEHAVIORAL OBJECTIVE 11 (POSTTEST)

Ratio Children Reaching Test Criterion	Percentage of Children Reaching Test Criterion	Test Criterion (Ratio Form)	Test Criterion (Percentage Form)
7/25	28	1/1	100

Tables 44 through 55 include results from the pretest, quiz(zes) and posttest for each behavioral object that was measured, both in a one dimensional and in a two dimensional sample space.

TABLE 44

BEHAVIORAL OBJECTIVE 3 (ONE DIMENSIONAL

Test	Number of Items	Ratio of Correct Responses	Percentage of Correct Responses
Pretest	3	44/75	58.7
Quiz I	2	29/50	58.0
Quiz II	2	46/48	95.8
Posttest	3	72/75	96.0

TABLE 45

BEHAVIORAL OBJECTIVE 3 (TWO DIMENSIONAL)

Test	Number of items	Ratio of Correct Responses	Percentage of Correct Responses
Pretest	4	1/100	1.0
Quiz IIB	2	37/48	77.1
Posttest	4	93/100	93.0

TABLE 46

BEHAVIORAL OBJECTIVE 4a (ONE DIMENSIONAL)

Test	Number of items	Ratio of Correct Responses	Percentage Correct Responses
Pretest	3	21/75	28
Quiz I	1	24/25	96
Posttest	3	74/75	98.8

TABLE 47

BEHAVIORAL OBJECTIVE 4a (TWO DIMENSIONAL)

Test	Number of Items	Ratio of Correct Responses	Percentage Correct Responses
Pretest	5	0/125	0
Quiz IIB	3	62/72	86.1
Quiz III	2	37/46	80.4
Posttest	5	125/125	100

TABLE 48

BEHAVIORAL OBJECTIVE 4b (ONE DIMENSIONAL)

Test	Number of Items	Ratio of Correct Responses	Percentage Correct Responses
Pretest	3	18/75	24
Quiz I	4	84/100	84.0
Quiz IIA	4	85/96	88.5
Quiz III	1	19/23	82.6
Posttest	3	73/75	97.3

TABLE 49

BEHAVIORAL OBJECTIVE 4b (TWO DIMENSIONAL)

Test	Number of Items	Ratio of Correct Responses	Percentage of Correct Responses
Pretest	5	3/125	2.4
Quiz IIB	3	43/72	59.7
Quiz III	2	36/46	78.3
Posttest	5	118/125	84.4

TABLE 50

BEHAVIORAL OBJECTIVE 4c (ONE DIMENSIONAL)

Test	Number of Items	Ratio of Correct Responses	Percentage of Correct Responses
Pretest	2	7/50	14.0
Quiz I	2	44/50	88.0
Quiz II (A and B)	2	47/48	97.9
Posttest	2	47/50	94.0

TABLE 51

BEHAVIORAL OBJECTIVE 4d (ONE DIMENSIONAL AND TWO DIMENSIONAL)

Test	Number of Items	Ratio of Correct Responses	Percentage of Correct Responses
Pretest	2	17/50	34.0
Quiz I	2	48/50	96.0
Quiz II (A and B)	2	47/48	97.9
Posttest	2	49/50	98.0

TABLE 52

BEHAVIORAL OBJECTIVE 5 (ONE DIMENSIONAL)

Test	Number of Items	Ratio of Correct Responses	Percentage of Correct Responses
Pretest	6	91/150	60.7
Exercise (7I)	12	261/300	87.0
Quiz IV	3	68/75	90.7
Posttest	6	148/150	98.7

TABLE 53

BEHAVIORAL OBJECTIVE 6 (TWO DIMENSIONAL)

Test	Number of Items	Ratio of Correct Responses	Percentage of Correct Responses
Pretest	4	29/100	29.0
Quiz IV	4	83/100	83.0
Posttest	4	90/100	90.0

TABLE 54

BEHAVIORAL OBJECTIVE 7 (ONE DIMENSIONAL)

Test	Number of Items	Ratio of Correct Responses	Percentage of Correct Responses
Pretest	5	44/125	35.2
Exercise 7I	5	108/125	86.4
Quiz IV	2	48/50	96.0
Posttest	5	118/125	94.4

TABLE 55

BEHAVIORAL OBJECTIVE 7 (TWO DIMENSIONAL)

Test	Number of Items	Ratio of Correct Responses	Percentage of Correct Responses
Pretest	3	20/75	26.7
Quiz IV	5	105/125	84.0
Posttest	3	68/75	90.7

TABLE 56

INDIVIDUAL PRETEST - POSTTEST RESULTS

(Number of Correct Responses on the 72 Item Test)

Subject (Student)	Pretest	Posttest	Subject (Student)	Pretest	Posttest
1	25	69	14	28	66
2	23	64	15	7	57
3	37	67	16	37	66
4	23	70	17	44	69
5	31	68	18	36	67
6	22	70	19	22	68
7	39	69	20	14	60
8	35	67	21	22	69
9	34	69	22	26	67
10	18	69	23	35	68
11	21	69	24	30	71
12	26	65	25	19	65
13	28	61			

APPENDIX D

BACKGROUND DATA OF STUDENTS

TABLE 57

STUDENT BACKGROUND DATA
(IOWA TESTS, MATH GRADES, IQ SCORES)

	Iowa Test of Basic Skills Form 4 (Percentile Scores)					Math Grade		Lorge-Thorndike IQ Test Level 3 Form A		
Subject (Students)	Reading Comprehension	Arithmetic Concepts	Arithmetic Problems	Total Arithmetic	Composite	1st Quarter	2nd Quarter	Verbal	Non-Verbal	Comp
1	92	62	49	59	76	C+	C	141	122	131
2	87	65	55	59	85	B+	B	122	113	117
3	51	75	96	91	56	New Student		109	102	106
4	86	76	94	89	85	A	A-	115	128	121
5	86	72	37	56	90	A-	D	117	103	110
6	66	79	88	84	88	C+	B-	115	111	113
7	95	83	96	93	91	C	C+	132	96	124*
8	71	84	96	93	78	B	B+	111	117	114
9	64	60	74	67	74	B+	B+	114	111	112
10	66	57	44	54	71	B-	B	115	117	116
11	75	88	89	90	82	B+	B+	121	128	124
12	97	74	76	75	90	B	A	128	120	124
13	75	76	49	65	74	B	B-	121	125	123
14	60	81	49	70	78	B+	B+	126	111	118*
15	62	57	68	65	74	D	F	106	109	107
16	62	89	85	87	76	B-	B	104	121	112
17	90	99	94	98	92	A-	B+	127	113	120*
18	73	36	30	32	56	B	B	125	116	120
19	94	88	81	86	95	B	B	131	117	124
20	33	41	89	70	43	D	D	109	104	106
21	98	74	89	84	92	B-	B	130	120	125
22	33	41	55	48	33	B	B	104	119	111
23	91	74	99	95	95	B	B+	127	126	126
24	73	67	85	77	76	B+	B	125	109	117
25	57	47	79	65	69	B-	B	126	118	122

*Administered March, 1969

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